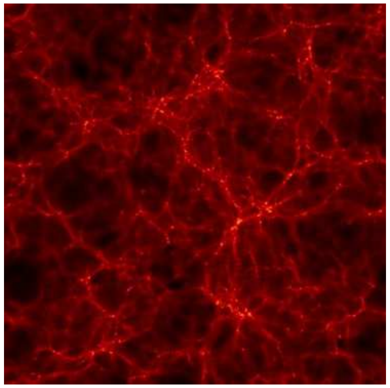

# Hierarchical Ansatz and Large Scale Structure Formation
A Very Brief non Mathematical Intro. without the agonising pain ;-)

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Books2read: Peebles,Peacock,Padmanabhan,Dodelson,Liddle,Weinberg

Review2check: Bernardeau et al. Physics Reports 2002

## Questions:

What is Hierarchical Ansatz?

What are various regimes in Gravitational clustering?

How much can be learned analytically from Euler-Poissson Equation?

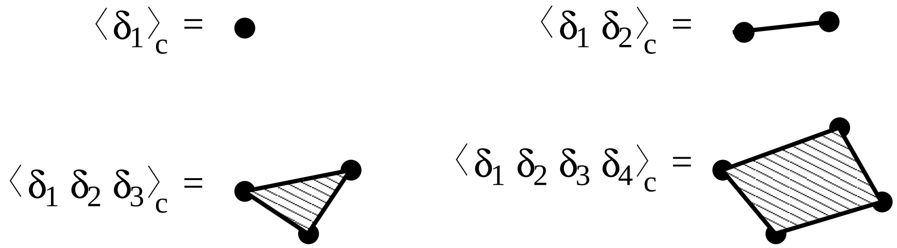
How do we treat non-linearity?

How do we describe non-Gaussianity?

What do we learn from Numerical simulations?

Has non-Gaussianity been measured from galaxy Surveys?

Are there any analytical somutions to Vlassov-Poisson's Equation?




## How do we describe Clustering?

Statistical Description using the Correlation Functions

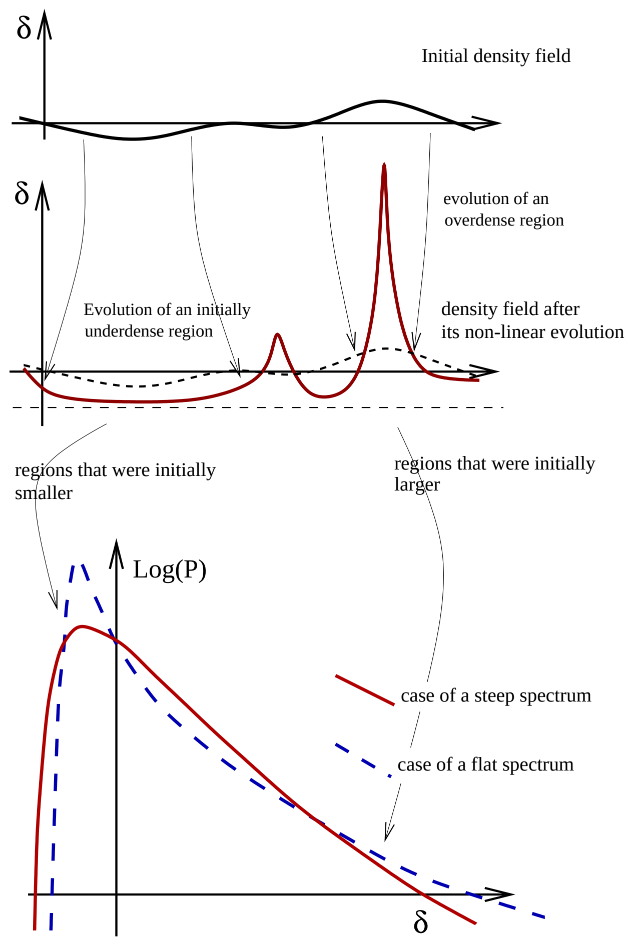
Gravity is non-linear which introduces non-Gaussianity

Multi-point Correlation Functions to describe non-Gaussianity.

Multi-point Correlation Functions in $k$-space or Multispectrum (Fry 1984).

All orders: Probability Distribution Function, PDF.

Geometrical Descriptors: Shape statistics of over(under)dense regions.
Topological Descriptors: Genus, Minkowski Functionals.


δ
Initial density field
δ
evolution of an
overdense region
Evolution of an initially
underdense region
density field after
its non-linear evolution
regions that were initially
smaller
regions that were initially
larger
Log(P)
case of a steep spectrum
case of a flat spectrum
δ

# Gravitational Dynamics

$$\triangle_{\mathbf{x}}\Phi = 4\pi G a^2 \rho_0 \delta; \qquad (1a)$$

$$\dot{\delta} + \frac{1}{a}\nabla_{\mathbf{x}}.\left((1+\delta)\mathbf{u}\right) = 0; \qquad (1b)$$

$$(a\mathbf{u})^{\cdot} + (\mathbf{u}\nabla_{\mathbf{x}})\mathbf{u} = -\nabla_{\mathbf{x}}\Phi \qquad (1c)$$

Eulerian Equations : Reference is fixed
Lagrangian Equations : Reference frame is moving with the fluid element.

Zeldovich Approximation: Linear order Lagrangian Theory

Evolution of phase space density: Vlassov Poisson Collisonless Dynamics.

Evolution of Higher order Correlation functions from BBGKY hierarchy

Numerical Simulations: PM, $\mathrm{P}^3$M, Tree, Adaptive and Hybrid approaches.

Simplified Dynamics or Approximations.

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## Perturbative regime

$$\delta(R,t) = \delta(R,t)^{(1)} + \delta(R,t)^{(2)} + \delta(R,t)^{(3)} \quad (1)$$
$$\theta(R,t) = \theta(R,t)^{(1)} + \theta(R,t)^{(2)} + \theta(R,t)^{(3)} \quad (2)$$

Here $\delta(R,t)^{(2)} \propto (\delta(R,t)^{(1)})^2$ and $\delta(R,t)^{(3)} \propto (\delta(R,t)^{(1)})^3$

Borrows results from Field Theory.
Assume the perturbations are small for the series to converge

Analysis is done mostly in Fourier domain

$$\langle \tilde{\delta}(\mathbf{k}_1,\tau)\tilde{\delta}(\mathbf{k}_2,\tau)\tilde{\delta}(\mathbf{k}_3,\tau)\rangle_c = \delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\ B(\mathbf{k}_1,\mathbf{k}_2,\tau), \quad (3)$$

$$\tilde{Q} \equiv \frac{B(\mathbf{k}_1, \mathbf{k}_2, \tau)}{P(k_1,\tau)P(k_2,\tau) + P(k_2,\tau)P(k_3,\tau) + P(k_3,\tau)P(k_1,\tau)}, \quad (4)$$

$\longrightarrow$ variance at a particular length scale is small

All qunatities are smoothed expanded in powers of variance at a given scale.

All qunatities are smoothed and expansion is in powers of variance at a given scale.

Ensemble average of various statistics are computed at different order.

Lowest order diagrams are called **Tree-diagrams**

Higher order contributions are called **loops** .

The simplest objects that can be computed are called cumulants.

They are normalised one-point moments $S_N = \langle \delta^N \rangle / \langle \delta^2 \rangle^{N-1}$.

At the next level $C_{pq} = \langle \delta_1^p \delta_2^2 \rangle / \langle \delta^2 \rangle^{p+q-1}$ are also well studied objects.

The results are derived using a specific form for the window. Tophat window provides simpler results

The $S_N$ parameters can be used to reconstruct the pdf in quasi-linear regime

$$P(\delta)\mathrm{d}\delta = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left(-\nu^2/2\right) \times$$
$$\times[1 + \sigma\frac{S_3}{6}H_3\,(\nu) + \sigma^2(\frac{S_4}{24}H_4\,(\nu) + \frac{S_3^2}{72}H_6\,(\nu))$$
$$+\sigma^3(\frac{S_5}{120}H_5(\nu) + \frac{S_4 S_3}{144}H_7(\nu) + \frac{S_3^3}{1296}H_9(\nu)) + ...]\mathrm{d}\delta, \qquad (5)$$

Here $H_n(\nu)$ are the Hermite polynomials

$$H_3(\nu) = \nu^3 - 3\nu, \qquad (6)$$
$$H_4(\nu) = \nu^4 - 6\,\nu^2 + 3, \qquad (7)$$
$$H_5(\nu) = \nu^5 - 10\,\nu^3 + 15\,\nu, \qquad (8)$$
$$\cdots$$

However a complete reconstruction can be performed at all order if we know the generating functions.
As opposed to the order by order approach described above.

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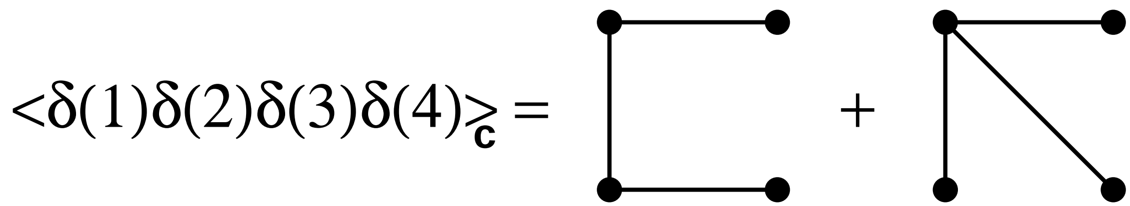


Figure 1: This figure shows two distinct topological diagrams contributing to tree level four point function. The left figure can be build up from two lower order twopoint diagrams.There are one new diagrams at each order and the hierarchy can not be closed without making any specific assumptions. The amplitude of the snake(left) diagram is denoted as $R_a = \nu_2^2$ and of the star (right) diagram is denoted as $R_b = \nu_3$. Ther will be an explosion of such diagrams at higher order which are mostly of mixed kind - neither snakes nor stars.


Figure 2: The loop diagrams are explaied in this figure. The tree diagrams are the dominant ones and each loop contributes a factor of $\langle\delta^{(1)}\rangle^2$. Tree-level and one loop corrections to two-point correlation functions are shown here. All loop corrections can be computed using PT. However such calculations can produce divergent results for certain spectra.

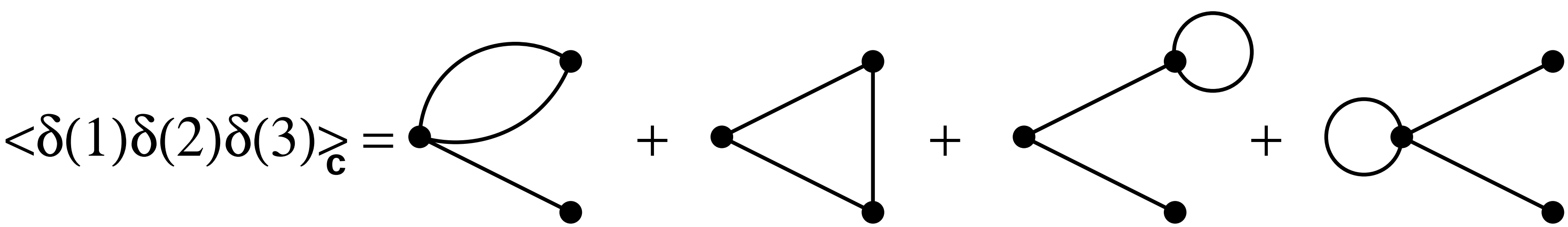


Figure 3: the tree and loop diagram contributions are depicted here for three point correlation function. The loops start to dominate as variance increases.

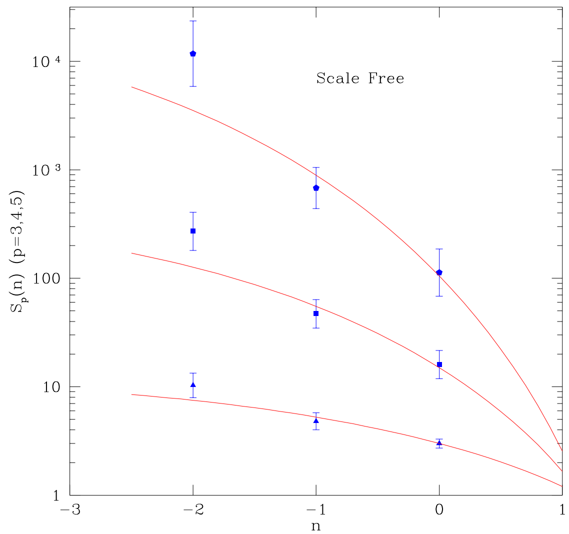


Figure 4: $S_N$ parameters are plotted as a function of spectral index $n$, Only scale free simulations $P(k) = Ak^n$ are being considered. The dots with error bars are results from simulations. Results for $N = 3, 4, 5$ are being shown. Results correspond to the perturbative regime. Error bars represent scatter among various realisations.

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## Non Perturbative Regime: Hierarchical Ansatz - 1

Gravitational clustering do not completely erases memory of initial conditions.

However it approaches a scale free regime at smaller scale.

$\xi_N(\lambda \mathbf{x_1}, \ldots \lambda \mathbf{x_n}) = \lambda^{-(N-1)} \xi_N(\mathbf{x_1}, \ldots, \mathbf{x_N})$

There are no characteristic time scale associated with $\Omega_M = 1$ Universe.

There are no scales in scale free initial condistions such as $P(k) = Ak^n$.

Stability of stable clustering h(n+D)= cosnt?

The correlation hierarchy admits a scale free behaviour.

Mathematical machinary from field-theory can produce meaningful insights.

All multi-point statistics is build from the two-point correlation functions

*Connected* multipoints gets contributions from all toplogical diagrams *snake*,and *stars*

Tree-Level structures remains intact but *renormalised* vertices

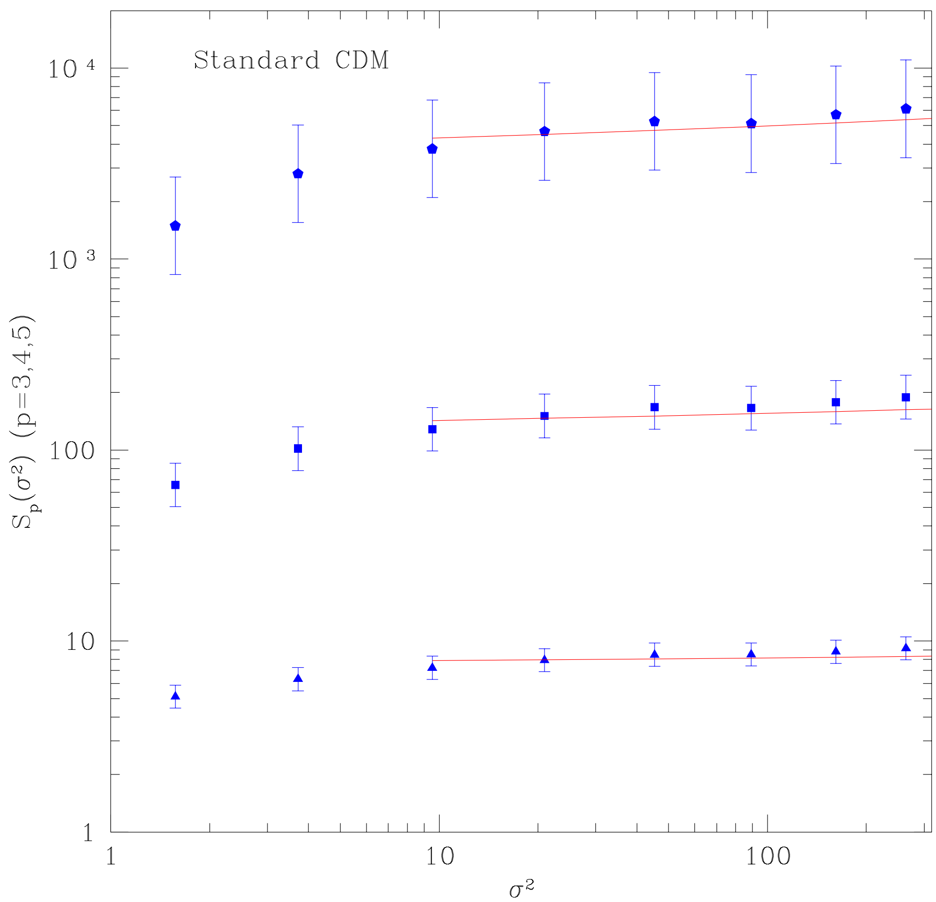


Figure 5: $S_N$ parameters are being plotted as a function of variance $\sigma^2(R)$. Low values fof $\sigma^2(R)$ correspond to quasilinear regime and higher values of $\sigma^2(R)$ correspond to highly non-linear regime. The red lines correspond to prediction from HEPT in the highly non-linear regime. Initial PS is that of SCDM.

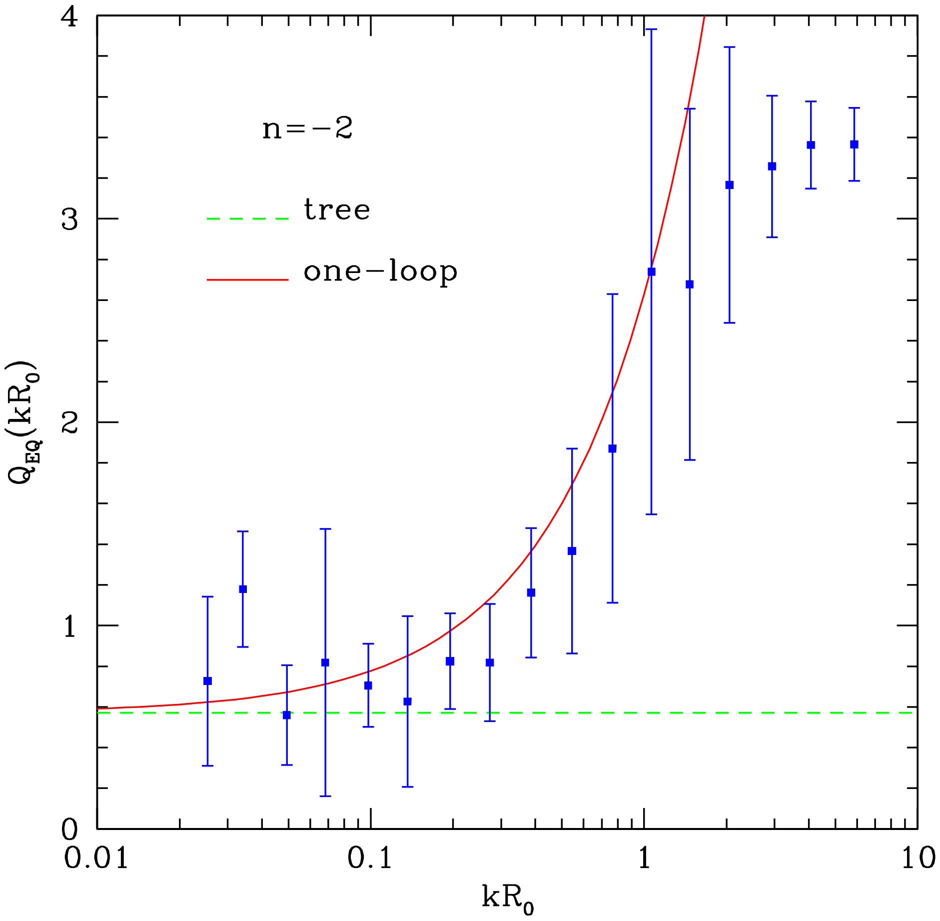


Figure 6: One loop and tree level bispectrum are plotted as a function of smoothing scales. The results are for equilateral triangular configurations.

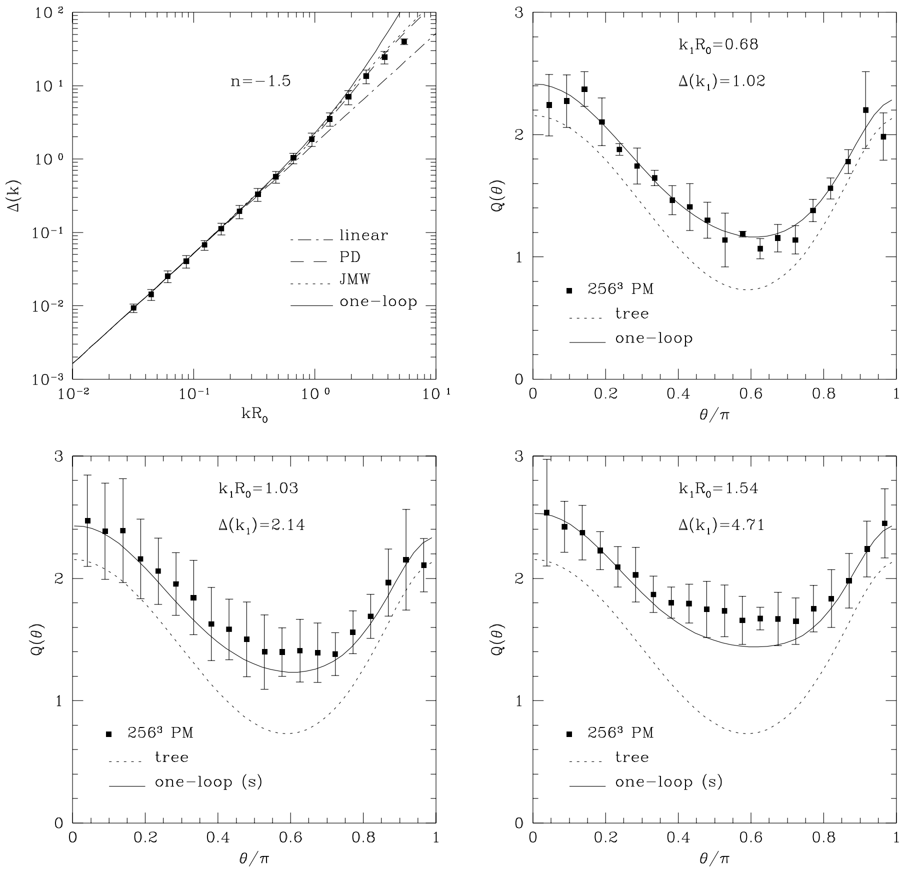


Figure 7: The left top panel shows the power spectra as a function of scale for a scale free initial conditions with $n = -1.5$. Various symbols depict results from numerical simulations. The reduced bi-spectrum Q for various triangular configuration is presented here. The triangle for computing the bispectrum has $k_1/k_2 = 2$. Q is ploted as a function of $\theta$ the angle between $k_1$ and $k_2$. Solid lines correspond to one-loop results and dashed lines correspond to tree-level diagrams.

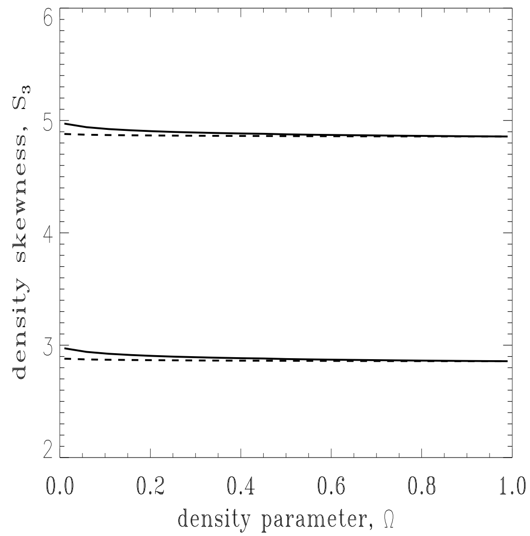


Figure 8: $S_3$ is plotted as a function of $\Omega_m$. For the solid curve $\Omega_\Lambda = 0$ and for the dashed curve $\Omega_m + \Omega_\Lambda = 1.$. Initial spectrum in both case is assumed to be a power law. Upper curves correspond to $n = -3$ and the bottom one $n = -1$. The results correspond to a top hat smoothing.

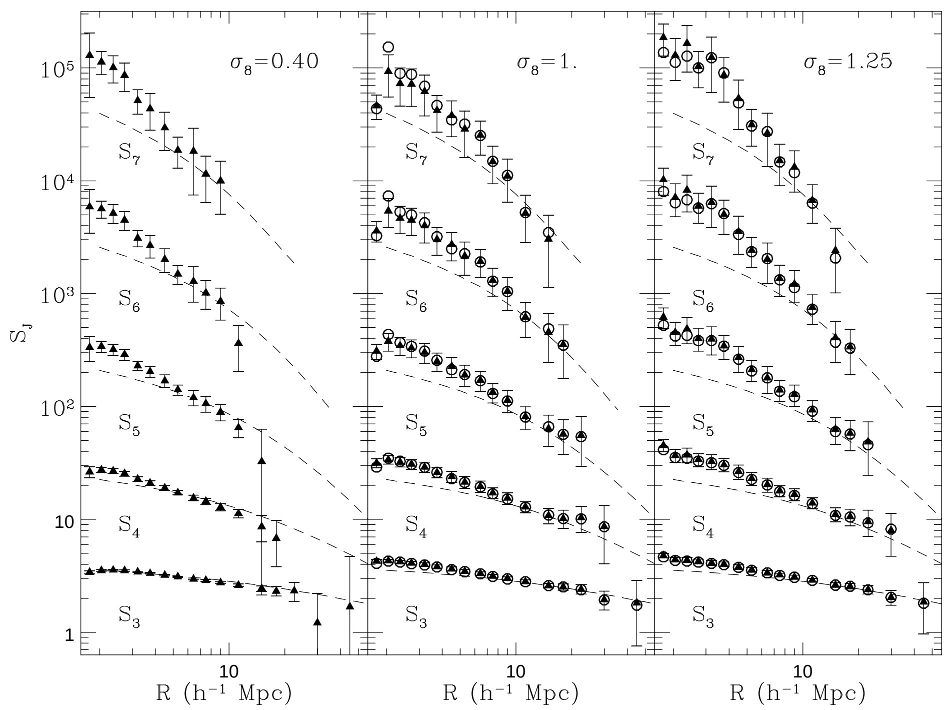


Figure 9: One loop and tree-level bi-spectrum is plotted for a equilateral configuration for $n = -2$. The dashed lines correspond to the tree level diagrams and the solid lines correspond to one loop corrections. Outputs from various time slices are being plotted from numerical simulations.

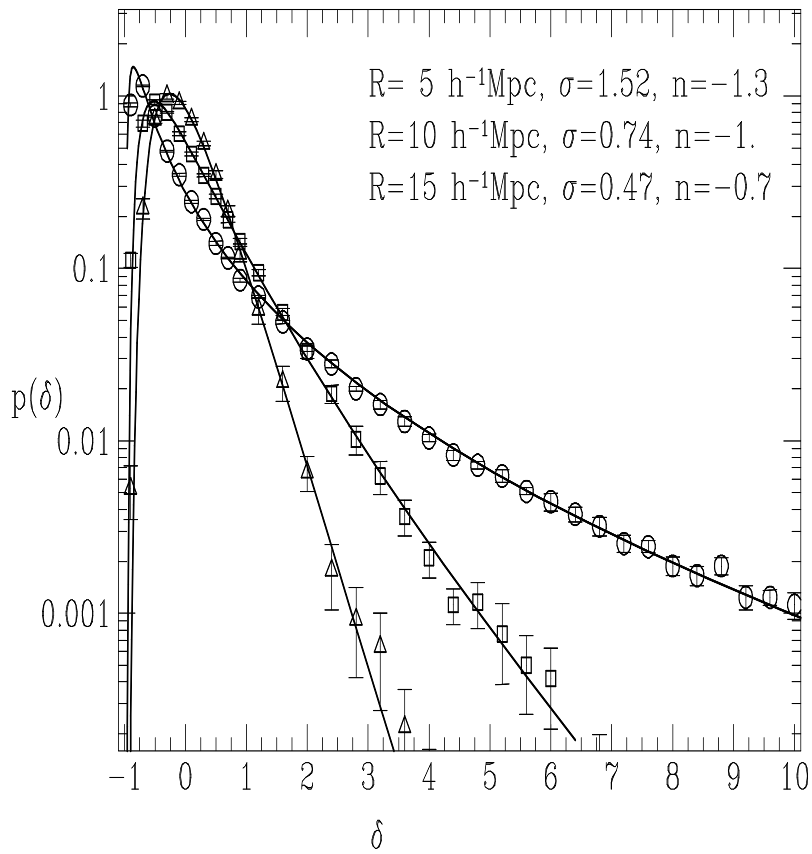


Figure 10: Analytical probability disribution functions are plotted as a function of smoothing radius $R$, initial power spectral index $n$ for scale free initial conditions. Symbols with error-bars are outputs from numerical simulations.

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## Non Perturbative Regime: Hierarchical Ansatz - 2

Vertex Generating Function $G_\delta(\tau) = \Sigma_n \frac{\nu_n}{n!}(-\tau)^n$
Void Probablity Function $\phi(y) = \Sigma_N (-y)^N \frac{S_N}{N!}$

Assumptions regarding $G_\delta(\tau)$ gives us a specific $\phi(y)$ and hence a pdf $P(\delta)$.

In perturbative regime $G_\delta(\tau)$ can be linked to initial conditions through gravitational dynamics.
However this is not possible in highly non-linear regime. Nevertheless there are well motivated ansatze.

## Intermediate Regime

No clear Analytical Model
Eulerian PT fails before the Lagarangian PT

There has been recent attempt to extend PT using running spectral slope. This method is also known as HEPT
Depends on the fact that at highly nonlinear regime all $Q_N$ tends to become shape independent and reaches a value which is same as the linear configuration in perturbative regime
It can be used to construct the entire pdf for all possible density contrasts

# Galaxy Clustering

Projected Catalogs, e.g. APM, Angular correlation functions.
Tree structures survives the projection effects
Redshift Surveys, 2dF, SDSS
Bias, Mizing of scale, redshift space distortions
Local, non-linear bias can be incorporated $\delta_g = b(\delta)$
Non-Gaussian initial conditions can also be analysed Error estimates include Poisson noise, Finite volume corrections
All of these can be analysed assuming a hierarchical ansatz

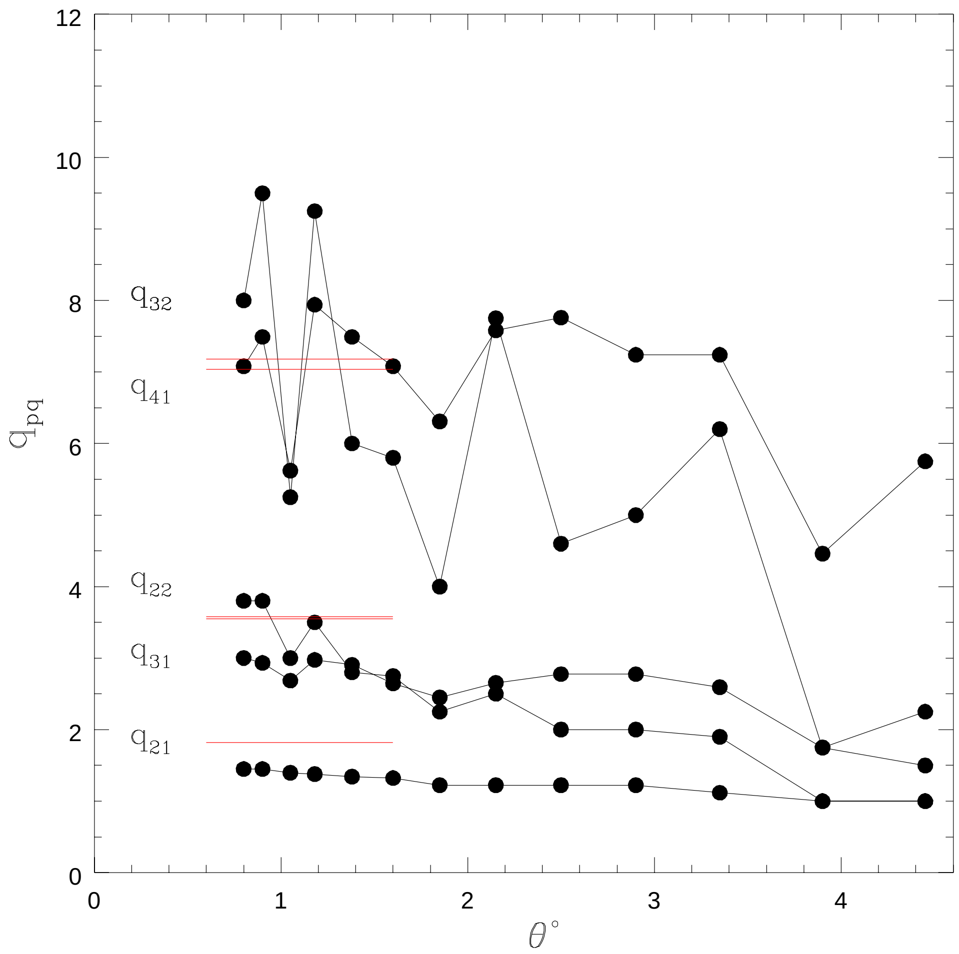


Figure 11: Analytical predictions of two-point cumulant correlators are plotted as a function of angular separation $\theta_0$. The results are for computations using the projected catalog APM. These results can be used to check the analytical predictions for $R_a$ and $R_b$.

## Statistics of Collapsed Objects

Assuming a correlation hierarchy for the underlying distribution means we can also predict the statistics of collapsed objects.

One can use them to study statistics of various cosmological objects. Galaxy Formation, Lyman-Alpha clouds, Warm absorbers, tSZ, ksZ and also 21cm observations .

This combines baryonic inputs with statistics derived from hierarchical ansatz

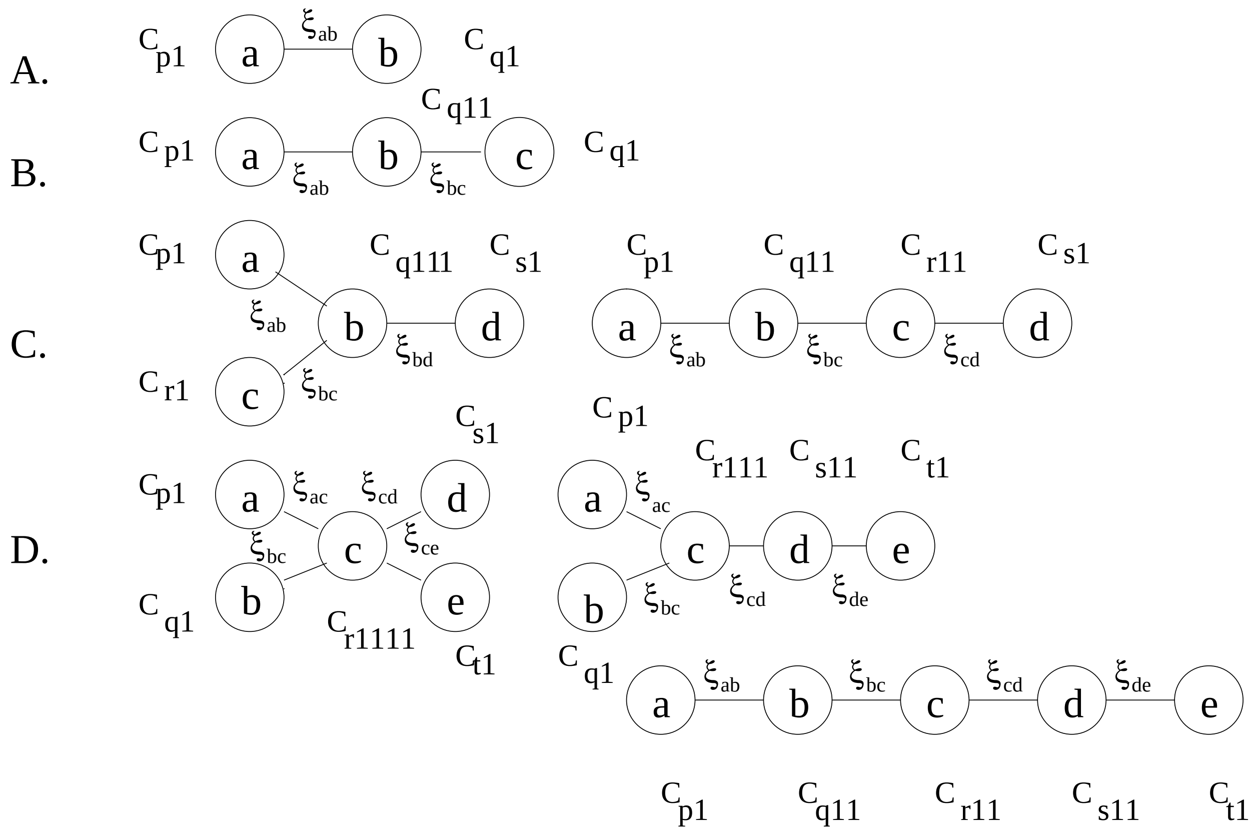


Figure 12: Overdense objects reproduces the same underlying hierarchy. However the vertices gets modified

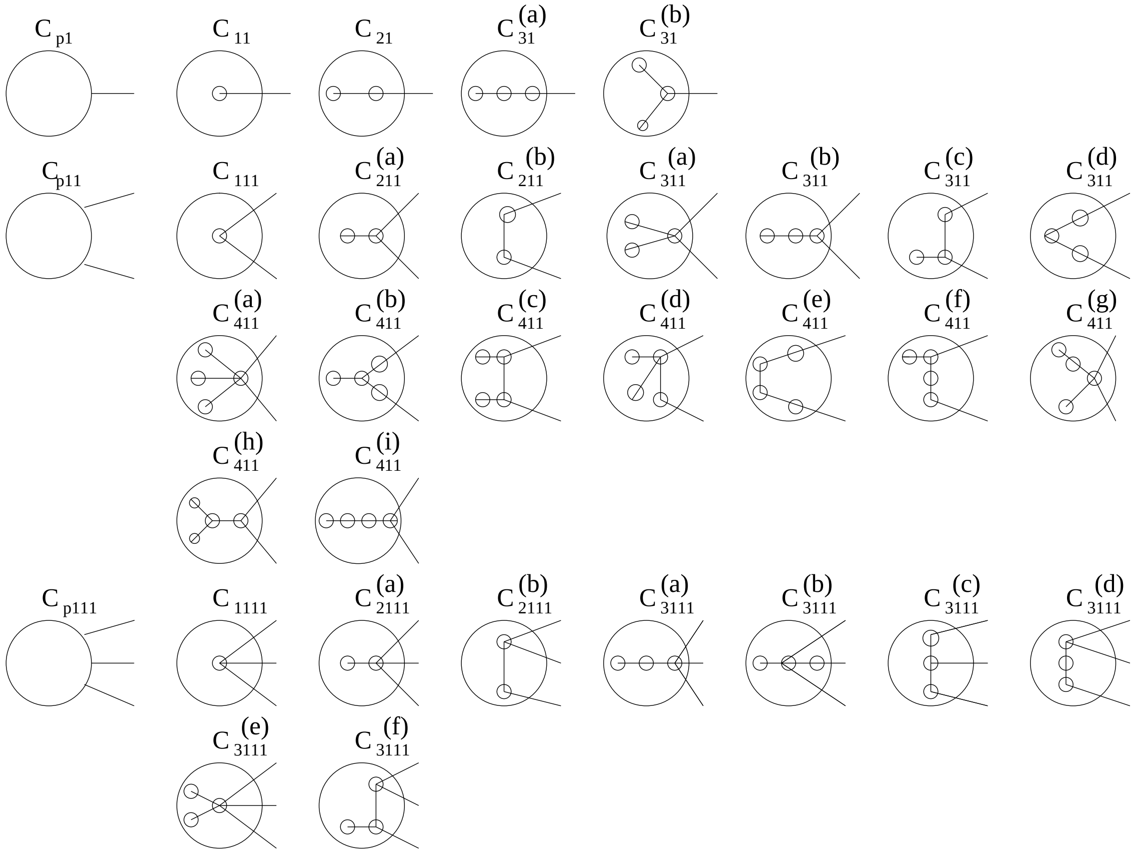


Figure 13: The new vertices can be reproduced in terms of the old vertices at the generating function level

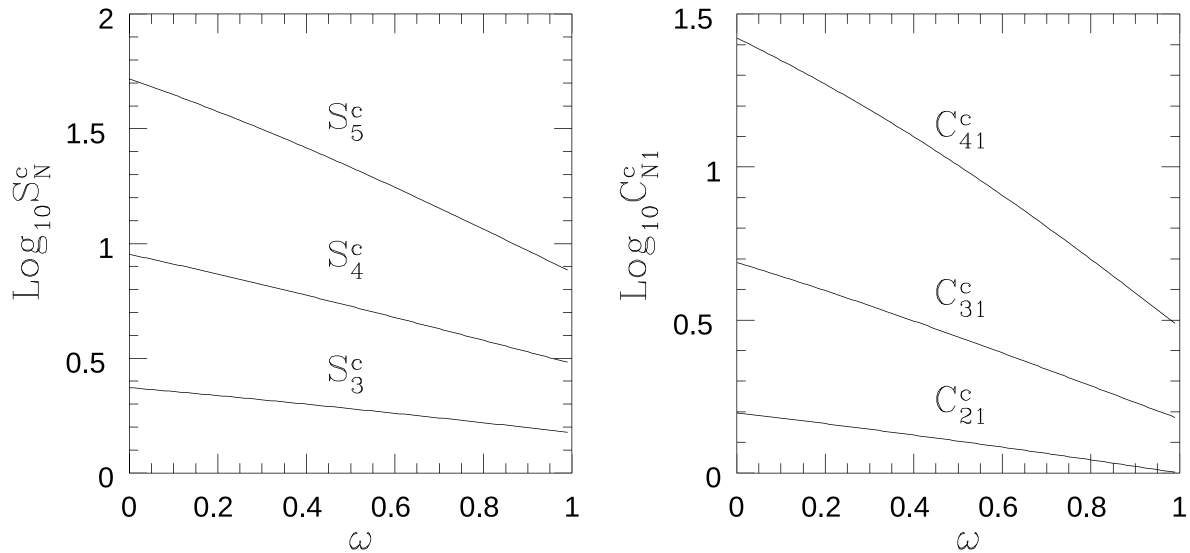


Figure 14: The moments of collapsed objects are however a function of the threshold.